readily accomplished. The irrational exponent, as in $a2^{\sqrt{2}}$, found unchallenged admission. It was natural to try exponents in the form of pure imaginary or of complex numbers (L. Euler, 1740). In the nineteenth century valuable interpretations were found which constitute the general theory of b^n where b and n may both be complex. Our exponential notation has been an aid for the advancement of the science of algebra to a degree that could not have been possible under the old German notation H, ξ, C, $\xi\xi$, ξC, bB, etc., or the other early notations. Nowhere is the importance of a good notation for the rapid advancement of a mathematical science exhibited more forcibly than in the exponential symbolism of algebra.

SOCRATES TEACHES MATHEMATICS

By Norman Anning,

Ann Arbor, Mich.

Socrates. What were we doing yesterday?

E: Played with blocks.

C: Made a cube.

A: Found what pieces make up $(x + y)^3$.

Socrates. Right. Shall we go on?

Chorus (reinforced from behind the scenes). Yes. Sure. Umhuh.

Socrates. Well, what do you see here?

H: Two blocks.

F: Two cubes.

C: One cube on another.

Socrates. Say it with algebra.

A: x^3+y^3.

B: The sum of two cubes.

Socrates. Why is it x^3+y^3?

D: Well, isn't it the X cube plus the Y cube?

Socrates. Why is it x^3+y^3?

A (after a pause during which Socrates, who is pulling hard, runs his finger significantly along the edge of the big cube and begins to write upon the sandboard "$x \times x \times x$") eagerly. Because if we call the edge of one x and of the other y that tells how much wood there is.

Since actual reproduction of the historic Socrates is difficult for even the skilled historian and for the amateur is quite impossible we choose the easier course of introducing into a modern classroom a being who shall bear the name and some of the (reputed) characteristics of the real Socrates.

Socrates (pointing to the blocks): Yes, you may call this the sum of two cubes and this (pointing to the finished x^3+y^3 on the sandboard) is our short way of writing down their total volume, or, as A puts it, a measure of all the wood in them, of all the space they occupy.

E (who has been gazing up through the branches): What's space?

Socrates. Don't you know yet what space is?

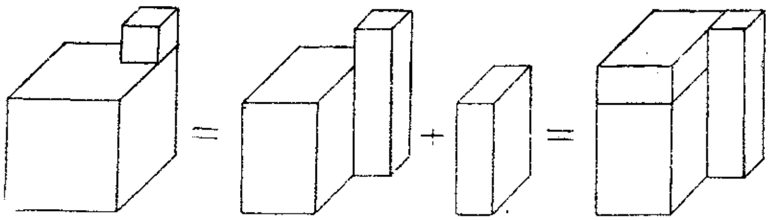


FIG. 1.

Socrates: Now if I cut this piece off and put it up here, is there any change in the volume?

Chorus: No.

H: Yes, the sawdust!

Socrates: Hush. What do you observe?

D: It fits.

B: It's all the same height.

F: It looks like two long blocks.

Socrates: Good. How high is it?

A: $x+y$.

Socrates: Now let's study F's two long blocks. B, take the stylus and write down our results. The small one first.

H: It's square on the end.

C: It is y by y by $(x+y)$.

Socrates: What is its volume?

A: The product of those numbers.

B writes down $(x+y)y^2$.

Socrates (ponders, perhaps on the question "Can a volume be a product?" but says nothing and the quest goes on). Now the larger block.

C: Well, it is $(x+y)$ high.

E: And it's x this way.

Socrates: Go on. How thick is it?

A: $(x-y)$.

B, instructed by the class, writes down $(x+y)x(x-y)$.

Socrates (turning to the sand-board). Can anyone see a way to improve that?

D: There's an $(x+y)$ in each part.

Socrates takes the stylus and writes $(x+y)$ $(x(x-y)+y^2)$ and finally $(x+y)$ (x^2-xy+y^2).

Socrates: Could the sum of any two cubes be handled similarly?

Class: Yes.

Socrates: And what would we have?

E (*sotto voice*): Another formula to learn.

Socrates: And worth learning because it is a gem of universal truth.

With some assistance they put it into words and, holding up their hands, march around chanting

"x^3+y^3, tra-la-la,
Always has a factor, ha-ha-ha!"

Socrates: What shall we try next?

A: The difference of two cubes.

D: How could you take one cube from another?

B: Get a chisel and gouge it out, you sompey.

H: It would be better to saw it out.

Socrates: Well, practical man, tell us how you would saw it out.

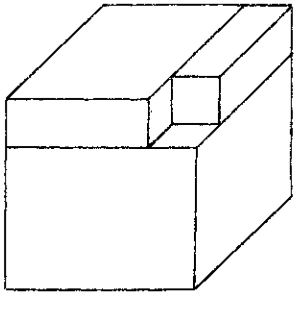
H: First I'd cut off a slab of the right thickness; from that I could cut a piece of the right width and then just saw the cube off the end.

Socrates: Do it.

H gets saw, straight-edge and pencil and goes to work. The other youths pretend to marvel at his "form."

Socrates (seeing an opportunity): Why does he draw lines on two faces of the block?

No answer. H finishes, takes the small cube for his reward and piles the rest as it was.



The boys attack the pile eagerly and, almost unaided, arrive at the final formula.

$(x^3-y^3)=(x-y)x^2+(x-y)xy+(x-y)y^2$.

A: Master, in a tablet of the ancients I find inscribed these symbols. $x^3+y^3+z^3-3xyz$. Could we study them tomorrow?

Socrates: If you will in the meantime cut a cube to exhibit $(x+y+z)^3$ we will gladly invoke the aid of the gods and discover what we can.

E: Could you make one cube out of two?

Socrates: I know not; I'll ask Theaetetus.

Socrates, suddenly remembering the parting words of Xantippe, hastens away to the market of the slayer of oxen and the youths remain in silence till he is out of earshot.

B (clapping the largest blocks together): Yea! Sock! but say, fellers, those rope-pullers of the Piraeus havent a snowball's chance to win in the Pan-Attic league.

C: I would burn incense to Zeus. Gimme a match.

Exeunt.

Postscript.

$(x+y+z)^3 = A+2B$.

Where $A=(x+y+z)\ (x^2+y^2+z^2)=x^3+y^3+z^3+\sum x^2+y$.

$B=(x+y+z)\ (yz+zx+xy)=3xyz+\sum x^2y$.

$A-B = (x+y+z)\ (x^2+y^2+z^2-yz-zx-xy) = x^3+y^3+z^3-3xyz$.

$A+2B=x^3+y^3+z^3+3\sum x^2y+6xyz$.

A good student should have little difficulty in showing that $A=xyz+(y+z)\ (z+x)\ (x+y)$, in finding three, or more, symmetrical ways of exhibiting $(x-y)\ (y-z)\ (z-x)$ and in extending the method to other interesting cubic identities.

PROBLEM DEPARTMENT

Conducted by J. A. Nyberg

Hyde Park High School, Chicago.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and solve problems here proposed. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. If you have any suggestion to make, mail it to him. Address all communications to J. A. Nyberg, 1044 E. Marquette Road, Chicago.

LATE SOLUTIONS

772. 774. *N. P. Pandya, Mehsana, N. G., India.*

SOLUTIONS OF PROBLEMS

771. *Proposed by L. E. Lunn, Heron Lake, Minnesota.*

Find the greatest numerical factor of $x(x^{12}-1)$ for positive integral values of x greater than 1.

Comment by B. F. Yanney, Wooster College, Ohio.

It seems to have been overlooked by the solver of problem 771, in the April number of SCHOOL SCIENCE AND MATHEMATICS, that if x is an odd integer, $x(x^4-1)=x(x-1)\ (x+1)\ (x^2+1)$ is divisible by 240, and hence $x(x^{12}-1)$ is divisible by 21840. The additional factor 2 may be accounted for thus: $(x-1)\ (x+1)$ is divisible by 8, since each factor